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Using ψ-Operator to Formulate a New Definition of Local Function

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Abstract: **In this paper, we use ψ-operator in order to get a new version of local function. The concepts of maps, dense, resolvable and Housdorff have been investigated in this paper, as well as modified to be useful in general**

الخلاصة: استخدمنا في هذا البحث العملية ابساي من أجل الحصول على نسخه جديده من الدالة المحلية.وقد تم تحقيق في مفاهيم الدوال, الكثافة, المجموعات القابلة للحل ولفضاء الهاوسدورف,وكذلك تعديلها لتكون مفيدة بشكل عام.

Keywords: **ψ-operator, dense, resolvable and Housdorff.**

1. INTRODUCTION AND PRILMINARIES

As requirements for our work, we define here the following concepts sequentially: Ideal space, local function, Kuratowski closure, dense, T*-dense, I-dense, codense, ψ-operator, resolvable, I-open, pre-I-open, scattered set and Housdorff space. We start define the ideal space. Let (X, T) be a topological space with no separation properties assumed. The topic of ideal topological space has been considered by (Kuratowski, 1966) and (Vaidyanathaswamy, 1960). An ideal I on a topological space (X, T) is a nonempty collection of subsets of X which satisfies the following two condition:

(1) If A I and B A, then B I (heredity).

(2) If A I and B I, then A B I (finite additivity).

Moreover a σ-ideal on (X, T) is an ideal which settle (1), (2) and the following condition:

3.If {Ai : i = 1,2,3,….} I, then {Ai : i =1,2,3,….} I (countable additivity).

An ideal space is a topological space (X, T) with an ideal I on X and is denoted by (X, T, I).For a subset A X, A*(I)={x X : U A I for every U T (x)} is called the local function of A with respect to I and T (Kuratowski, 1933).We simply write A* instead of A*(I) in case there is no chance for disorder. It is familiar that Cl*(A) = A A* defines a Kuratowski closure operator for a topology T *(I) which is finer than T. During this paper, for a subset A X, Cl (A) and In (A) indicate the closure and the interior of A ,respectively. A subset A of an ideal space (X, T, I) is said to be dense (resp, T *-dense (Dontcher, Gansster and Rose, 1999), I –dense (Dontcher, Gansster and Rose,1999) if Cl (A) = X (resp Cl*(A) = X, A*= X).An ideal I on a space (X, T) is said to be codense (Devid, Sivaraj and Chelvam, 2005) if and only if T I = {Φ}.For an ideal space (X, T, I) and for any A X, where I is codense. Then: dense, T*-dense and I-dense are comparable (Jankovic and Hamlett, 1990). (Natkanies, 1986) used the idea of ideals to define another operator known as ψ-operator elucidates as follow: For a subset A X, ψ (A) = X-(X-A)*.Equivalently ψ(A) = {M T: M-A I}.It is obvious that ψ(A) for any A is a member of T. For an ideal space (X, T, I) and Y X then (Y, T y, I y) is an ideal space where T y = {U Y: U T} and I y = {U Y: U I} = {U I: U Y}. In 1943, Hewitt put forward the opinion of a resolvable space as follows: A nonempty topological space (X, T) is said to be resolvable (Hewitt, 1943) if X is the disjoint union of two dense subsets. Given a space (X, T) and A X, A is called I -open (Jankovic and Hamlett, 1990)(resp per- I –open(Donkhev,1996)if A In (A*)(resp A In Cl*(A).A set A X is called scattered (Jankovic and Hamlett, 1990) if A contains no nonempty dense-in-itself subset. A space (X, T, I) is called I -Housdorff (Dontcher,1995) if for each two distinct points x ≠ y, there exist I -open sets U and V containing x and y respectively, such that U V=Φ. Throughout this paper we define a new local function and generalizations .Many characterizations, proprieties and relation between them are obtained.

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2. ()Ψ*-OPERATOR

In this section we put forward a new type of local function by using ψ-operator, in order to do this we have to have a deep looking in (Jankovic and Hamlett, 1992) work of the local function in ideal space.

Definition 2.1

Let (X, T, I) be an ideal space. An operator (.)ψ*: P(x) → P(x), called ψ -local function of A with respect to I and T, is define as follow: for any A X, Aψ* (I, T)={x X: ψ (U) A I, for every open subset U T(x) }, where T(x)={U T,x U}. When there is no chance for confusion Aψ* (I, T) is briefly denoted by Aψ*.

Fact

Let (X, T, I) be an ideal space .Then A* Aψ*, for any A X.

Now we define a new closure operator in terms of ψ -local function.

Definition 2.2

Let (X, T, I) be an ideal space. For any A X, we define

(1) Cl ψ (A) = {x X: ψ (U) A ≠ Φ for every U X, x ψ (U)}. It is clear that Cl (A) Cl ψ (A).

(2) Cl ψ*(A) (I, T) = A Aψ*

When there is no chance for confusion Cl ψ*(A) (I, T) is briefly denoted by Cl ψ*(A).

Note

Let (X, T, I) be an ideal space. Then Cl ψ*(A) Cl ψ (A), for any A X.

We discuss the properties of ψ -local function in following theorem:

Theorem 2.3

Let (X, T) be a topological space, I and J be two ideals on X, and let A and B be two subsets of X. Then the following properties hold:

1. If A B, then Aψ* Bψ*.

2. If I J, then Aψ* (J) Aψ* (I).

3. Aψ* =Cl (Aψ*) Cl ψ (A).

4. (Aψ*)ψ* Aψ*.

5. (A B)ψ* = Aψ* Bψ*

6. (A B)ψ* Aψ* Bψ*

7. For every I ϵ I, then (A I)ψ* = Aψ* = (A \ I)ψ*.

8. If G T, then G Aψ* (G A)ψ*.

9. If U T, then U Cl ψ* (A) Cl ψ* (U A).

10. If A I, then Aψ* = Φ.

Proof.

Straight from Definition2.1 and Definition 2.2

3. Ψ - DENSE AND GENERALIZATIONS

Definition 3.1

Let (X, T, I) be an ideal space, A X is called:

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(1) ψ -dense, if Cl ψ (A) = X.

(2) ψ -T*-dense, if Cl ψ*(A) = X.

(3) ψ -I-dense, if Aψ* = X.

The following remark is immediate from above definitions:

Remark 3.2

(1) If A is ψ -I-dense, then A is ψ -T*-dense.

(2) If A is ψ -T*-dense, then A is ψ -dense.

In the following example we show that the convers of above remark does not hold:

Example 3.3

Let X = {a, b, c}, T = {Φ, X, {a}, {b}. {a, b}} and I = {Φ, {b}}. If A = {a, b}, then Aψ* = {a, c} but Cl ψ* = X.

The study of ideal got new dimension when Codense ideal [10] has been incorporated in ideal space. In the following definition we introduce a new concept of condense by using ψ-operator.

Definition 3.4

Let (X, T, I) be an ideal space, an ideal I is called ψ -codense if, ψ (T) I = {Φ}.

Lemma 3.5

Let (X, T, I) be an ideal space and for every subset A X. If A Aψ*, then the following relations hold:

Aψ* = Cl (Aψ*) = Cl ψ (A) = Cl ψ*(A).

Proof.

Clear from Definition 2.2: (1), and Theorem 2.3: (3).

Theorem 3.6

Let (X, T, I) be an ideal space, and for every subset A X. If I is ψ -codense, then the following relations hold:

(1) Where A is ψ -dense, then it is ψ -T*-dense.

(2) Where A is ψ -T*-dense, then it is ψ -I-dense.

Proof.

This is an immediate consequence of Lemma 3.5.

The following theorem related to ψ -codense ideal:

Theorem 3.7

If X is finite, then X=X* if and only if, I is ψ -codense.

Proof.

Necessity. By Theorem 2.5: [1].we get that T I= {Φ}. Since ψ(T) I T I. It follow that ψ(T) I = {Φ}.So I is ψ -codense.

Sufficiency. Assume that I is ψ -codense. Since X-X* is open, let x X-X*. Then there exists Ux X-X*, and x X*.Then there exists Vx T(x), such that Ux Vx I. Assume that Wx = Ux Vx I. But X-X* = Wx. Hence X-X* I and ψ(T) I ≠ {Φ}, a contradiction. Therefore, X = X*.

Corollary 3.8

Let X be any nonempty set .If (X, T, I) is an ideal space where I is σ-ideal, then the following statement are equivalent:

(1) X = X*;

(2) T I = {Φ};

(3) ψ(T) I = {Φ}.

Proof.

Similar to the proof of Theorem 3.7

By Remark 3.2, and Theorem 3.6, the following relations hold:

ψ -dens ⇄ ψ -T*-dense ⇄ ψ -I-dense.

*: I is ψ -codense.

By Theorem 2.3 :(2), we have the following remark:

Remark 3.9

(1)Let (X, T, I) and (X, T, J) be two ideal spaces, I J, if A is ψ -I-dense with respect to J ,then A is ψ -I-dense with respect to I for any A X.

(2) Let (X, T, I) and (X, σ, I) be two ideal spaces, T σ, if A is ψ -I-dense with respect to σ ,then A is ψ -I-dense with respect to T for any A X.

By Remark 3.2 and Remark 3.9: (1), (2) we get the following corollary:

Corollary 3.10

(1)Let (X, T, I) and (X, T, J) be two ideal spaces, I J, if A is ψ -T*-dense (ψ -dense) with respect to J, then A is ψ -T*-dense (ψ -dense) with respect to I for any A X.

(2) Let (X, T, I) and (X, σ, I) be two ideal spaces, T σ, if A is ψ -T*-dense (ψ -dense) with respect to σ, then A is ψ -T*-dense (ψ -dense) with respect to T for any A X.

Definition 3.11

A function f: (X, T, I) → (Y, σ) is called ψ -map if, f (Cl ψ (A)) Cl f (A), for any A X.

Note

Where f:(X,T,I)→(Y,σ,f(I)) is a bijection map ,then f(I) will be an ideal on Y.

Theorem 3.12

If f: (X, T, I) →(Y, σ) is bijection ψ - map .Then the image of any ψ -dense subset A of X is dense in Y.

Proof.

Assume that A is ψ -dense in X. Since f is ψ -map, then f (Cl ψ (A)) Cl (f (A)).Implies that f (X) Cl (f (A)) and since f is surjection .Therefore, Y= Cl (f (A)). Hence f (A) is dense in Y.

Definition 3.13

A function f:(X,T,I)→(Y,σ,J) is called :

(1) ψ -(I,J) map ,if f (Cl ψ* (A)) Cl* f (A).

(2) ψ 0 - (I,J) map ,if f (Aψ*) (f(A))*.

(3) ψ 1- (I,J) map, if f (Aψ*) (f(A))ψ* .

(4) ψ 2- (I,J) map ,if f (Cl ψ* (A)) Cl ψ* f(A).

(5) ψ 3- (I,J) map, if f(Cl ψ(A)) Cl ψ f(A).

For any A X.

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From the above definitions we get the following remark :

Remark 3.14

(1) ψ 0 - (I, J) map → ψ - (I, J) map.

(2) ψ 1 - (I, J) map → ψ 2 - (I, J) map → ψ 3 - (I, J) map.

Theorem 3.15

(1) If f: (X, T, I) →(Y, σ) is bijection ψ 0-(I, J) map .Then the image of any ψ -I-dense subset A of X is f(I)-dense in Y.

(2) If f: (X, T, I) → (Y, σ) is bijection ψ - (I, J) map. Then the image of any ψ -T*-dense subset A of X is T*-dense in Y.

Proof.

(1)Assume that A is ψ -I-dense in X. Since f is ψ 0-(I, J) map, then f (Aψ*) (f (A))*. Implies that f(X) (f (A))*, and since f is surjection .Therefore, Y= (f (A))*.Hence f (A) is f (I)-dense in Y.

(2) Assume that A is ψ –T*-dense in X. Since f is ψ -(I, J) map, then f (Clψ*(A)) Cl* f (A)Implies that f(X) Cl* f (A), and since f is surjection .Therefore, Y= Cl* f (A).Hence f (A) is T*-dense in Y.

Theorem 3.16

(1)If f: (X, T, I) → (Y, σ, J) is ψ 1 - (I, J) bijection map. Then the image of any ψ -I-dense subset of X is ψ -f (I)-dense in Y.

(2) If f: (X, T, I) → (Y, σ, J) is bijection ψ 2-(I, J) map. Then the image of any ψ -T*-dense subset of X is ψ -T*-dense in Y.

(3) If f: (X, T, I) → (Y, σ, J) is bijection ψ 3-(I, J) map. Then the image of any ψ -dense subset of X is ψ -dense in Y.

Proof.

(1) Assume that A is ψ-I-dense in X. Since f is ψ 1-(I, J) map then f (Aψ*) (f (A))ψ*,and since f is surjection .Therefore, Y = (f (A))ψ*.Hence f (A) is ψ -f (I)-dense in Y.

(2) Assume that A is ψ –T*-dense in X. Since f is ψ 2-(I, J) map then f (Clψ*(A)) Cl ψ* f(A), and since f is surjection .Therefore, Y=Cl ψ*f(A).Hence f (A) is ψ –T*-dense in Y.

(3) Assume that A is ψ-dense in X. Since f is ψ 3-(I, J) map then f(Cl ψ(A)) Cl ψ f(A) and since f is surjection .Therefore, Y=Cl ψ f(A).Hence f(A) is ψ-dense in Y.

4. Ψ -RESOLVABILITY AND GENERALIZATIONS

In this section we define ψ- resolvable space and recall some of the generalizations of ψ-resolvable and their relationship with Housdorff space.

Definition 4.1

A nonempty topological space (X, T) is called:

(1) ψ -resolvable, if X is the disjoint union of two ψ -dense subsets.

(2) ψ-T*-resolvable, if X is the disjoint union of T*-dense and ψ-dense subsets.

(3) ψ-I-resolvable, if X is the disjoint union of I-dense and ψ -dense subsets.

It is clear that: ψ -I-resolvable → ψ -T*-resolvable → ψ -resolvable.

Theorem 4.2

Let (X, T) be a nonempty topological space, where I is ψ-codense. Then if X is ψ -T*-resolvability, then X is ψ -I-resolvability.

Question

We claim that ψ-resolvability does not imply ψ-T*-resolvability, but we could not find an example.

From Corollary 3.9 we get the following remark:

Remark 4.3

Let (X, T, I) , (X, T, J) be an ideal spaces, with I J, if X is ψ -resolvable with respect to J ,then X is ψ -resolvable with respect to I.

Definition 4.4

A function f:(X,T,I)→(Y,σ,J) ,is said to be bi*-map if :

(1) f: (X,T,I)→(Y, σ) is ψ 3-(I,J) map;

(2) f: (X,T*)→(Y, σ*) is a continuous map.

Theorem 4.5

Let f :(X, T, I) → (Y, σ) be bijection bi*- map, and if X is ψ -T*-resolvable space .Then Y is ψ -T*-resolvable with respect to the ideal f (I).

Proof.

Assume that X is ψ -T*-resolvable space, then there exists two disjoint ψ -dense and T*-dense subsets such that Cl ψ (A) = Cl*(B) = X. It is clear by Corollary 3.16, and Definition 4.4 that Y is ψ -T*-resolvable.

Lemma 4.6

If U is an open set and D ψ* = X, then U (D ψ (U))ψ*.

Proof

Let x U, and suppose if possible that x (D ψ (U))ψ* Then (D ψ (U))   ψ (V) I, and since ψ (U)   ψ (V) T. Therefore, x D ψ* = X. This is a contradiction .That is U (D ψ (U))ψ*.

Theorem 4.7

(1) Let U T and Cl ψ*(D) = X, then U Cl ψ* (D ψ (U)).

(2) Let U T, and Cl ψ (D) = X. Then U   Cl ψ (D ψ (U)).

Using Remark 3.2: (1), (2), and similar to the proof of Lemma 4.6.

Theorem 4.8

Let Y T, and Aψ* = X. Then A1ψ y* = Y, where A1 = A y.

Proof.

By lemma 4.6 Y (A Y)ψ x* = A1ψ x*.

Then Y = A1ψ x* y = A1ψ y*.

Using Remark 3. 2 :(1) ,(2), and Theorem 4.8we get the following theorem:

Theorem 4.9

(1) Let Y T, and Cl ψ*(A) = X. Then Cl ψ*y (A1) = Y, where A1 = A Y.

(2) Let Y T, and Cl ψ (A) = X. Then Cl ψ y (A1) = Y, where A1 = A Y.

Theorem 4.10

(1) Let (X, T, I) be ψ -I-resolvable space, if Y is open .Then (Y, Ty, I y) is ψ –I y- resolvable.

(2) Let (X, T, I) be ψ -T*-resolvable space, if Y is open .Then (Y, Ty, I y) is ψ -T*y- resolvable.

(3) Let (X, T, I) be ψ -resolvable space, if Y is open .Then (Y, Ty, I y) is ψ - resolvable.

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Proof.

(1) Assume that (X, T, I) is ψ -I-resolvable space .Then there exists two disjoint ψ -dense, and I-dense subsets such that: A B = X, and Cl ψ (A) = B* = X. Note that Y = Y X = Y (A B) = (Y A) (Y B) , and (Y A) (Y B) = Y (A B) = Φ. Put A1 = A Y and B1 = B Y. Then Y = A1 B1, A1 B1 = Φ. By corollary 3.10: (2), we have Cl ψ y (A1) = Y, and by lemma we have B*1y = Y. Hence (Y, Ty, I y) is ψ –I y-resolvable.

(2) Assume that (X, T, I) is ψ –T*-resolvable space .Then there exists two disjoint ψ -dense, and T*-dense subsets such that: A B = X, and Clψ (A) = Cl*(B) = X. Note that Y = Y X = Y (A B) = (Y A) (Y B) , and (Y A) (Y B) = Y (A B) = Φ. Put A1 = A Y and B1 = B Y. Then Y = A1 B1, A1 B1 = Φ. By Theorem 2.3.20: (3), we have Clψ y (A1) = Y, Cl*1y = Y. Hence (Y, Ty, I y) is ψ –T* y-resolvable.

(3) Assume that (X, T, I) is ψ -resolvable space .Then there exists two disjoint ψ –dense subsets such that: A B = X, and Clψ (A) = Clψ (B) = X. Note that Y = Y X = Y (A B) = (Y A) (Y B) , and (Y A) (Y B) = Y (A B) = Φ. Put A1 = A Y and B1 = B Y. Then Y = A1 B1, A1 B1 = Φ. By Theorem 2.3.20: (3), we have Clψ y (A1) = Clψ y (A1) = Y. Hence (Y, Ty, I y) is ψ – Ty - resolvable.

Definition 4.11

In ideal space (X, T, I) A subset A is called ψ -pre-open if A In Cl ψ (A).

Definition 4.12

(1) An ideal space (X, T, I) is called ψ -I -Housdorff, if for every distinct point x, y X, there exist two disassociate ψ -pre-open, and I-open sets containing each respectively.

(2) An ideal space (X, T, I) is called ψ -T*-Housdorff, if for every distinct point x, y X, there exist two disjoint ψ -pre-open, and pre-I-open sets containing each respectively.

(3) An ideal space (X, T, I) is called ψ -Housdorff, if for every distinct point x, y X, there exist two disjoint ψ -pre-open sets containing each respectively.

Lemma 4.13

If ({x})ψ I. Then ({x})ψ* = Φ.

Proof.

Assume that ({x})ψ* ≠ Φ, then there exist y ({x})ψ* such that ψ (U) {x} I for every U T(x). Implies that ({x})ψ I. This is a contradiction.

Theorem 4.14

(1) If an ideal space (X, T, I) is ψ -I-resolvable, and the scattered set of (X, T) are in I, then (X, T, I) is ψ -I-Housdorff.

(2) If an ideal space (X, T, I) is ψ -T*-resolvable, and the scattered set of (X, T*) are in I, then (X, T, I) is ψ -T*-Housdorff.

(3) If an ideal space (X, T, I) is ψ -resolvable, and the scattered set of (X, T) are in I, then (X, T, I) is ψ -Housdorff.

Proof.

(1) Let A and B be disjoint ψ -dense, and I-dense subsets of X such that X = A B. Since Clψ (A) = X, B* = X. Therefore, A In Clψ (A) and B In B*, then A and B are ψ -pre-open and I-open sets respectively. Let x, y be any two element of X and both x and y are in A .Take V = A\{y} and U = B {y}. It is easily observed (see [6], Theorem 2.3 (h)]) that V also I-open set and y in V. Now to show that U = A {x} is ψ -pre-open. Clψ (A {x}) = Clψ (A) Clψ ({x} = X Clψ ({x}) = X .Hence A {x} In Clψ (A {x}), so U is ψ -pre-open. Hence (X, T, I) is ψ -I-Housdorff.

(2) Let A and B be disjoint T*-dense, and ψ-dense subsets of X such that X = A B. Since Cl* (A) = X and Clψ (B) = X. Therefore, A In Cl* (A) and B In Clψ (B), then A and B are pre- I-open and ψ -pre-open sets respectively. Let x, y be any two element of X and both x and y are in A .Take V = A\{y} and U = B {y}. To show that V also pre- I-open set Cl*(A\{y}) = Cl*(A) \ Cl* ({y}) = X. Hence A\{y} In Cl*(A\{y}.Therefore, V is pre-I-open. Now to show that U = B

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{y} is ψ -pre-open. Clψ (B {y}) = Clψ (B) Clψ{y} = X Clψ ({y}) = X .Hence B {y} In Clψ (B {y}), so U is ψ -pre-open. Hence (X, T, I) is ψ –T*-Housdorff.

(3) Let A and B be disjoint ψ -dense subsets of X such that X = A B. Since Clψ(A) = Clψ(B) = X. Therefore, A In Clψ(A) and B In Clψ(B), then A and B are ψ-pre-open sets. Let x, y be any two element of X and both x and y are in A. Take U = A\{y} and V = B {y}. By Theorem 2.1.5: (8) we have V and U are ψ-pre-open sets. To show that U is ψ-pre-open, Clψ (A\{y}) = Clψ (A)\ Clψ({y}) = X\ Clψ({y}) = X, A\{y} In Clψ(A). Now to show that V is ψ-pre-open. Clψ(B {y}) = Clψ(B) Clψ({y}) = X, B {y} In Clψ(B). Hence U,V are ψ-pre-open. Therefore, (X, T, I) is ψ-Housdorff.

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